The strong and weak holographic principles

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ABSTRACT

We review the different proposals which have so far been made for the holographic principle and the related entropy bounds and classify them into the strong, null and weak forms. These are analyzed, with the aim of discovering which may hold at the level of the full quantum theory of gravity. We find that only the weak forms, which constrain the information available to observers on boundaries, are implied by arguments using the generalized second law. The strong forms, which go further and posit a bound on the entropy in spacelike regions bounded by surfaces, are found to suffer from serious problems, which give rise to counterexamples already at the semi-classical level. The null form, proposed by Fischler, Susskind, Bousso and others, in which the bound is on the entropy of certain null surfaces, appears adequate at the level of a bound on the entropy of matter in a single background spacetime, but attempts to include the gravitational degrees of freedom encounter serious difficulties. Only the weak form seems capable of holding in the full quantum theory.

The conclusion is that the holographic principle is not a relationship between two independent sets of concepts: bulk theories and measures of geometry vrs boundary theories and measures of information. Instead, it is the assertion that in a fundamental theory the first set of concepts must be completely reduced to the second.

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Contents

1 Introduction

The purpose of this paper is to examine the question: *"Exactly what consequences do the holographic principle[1, 2, 3], and the related entropy bounds[4], have for the construction of the quantum theory of gravity?"*. We will take it as given that a quantum theory of gravity should be both background independent and cosmological. This is because both background dependence and boundaries are almost certainly artifacts of approximations which, while convenient for certain purposes, exclude significant aspects of the problem of constructing a theoretical framework which includes and extends the principles of both relativity and quantum theory[5].

The question is not easy because most of the results which are so far known to bear on it are concerned with semiclassical approximations or weak coupling limits. Many are also limited to situations with boundaries, either asymptotic or finite. It is, of course, always possible that the holographic principle is only a characterization of the semiclassical theory, perhaps because it is no more than a re-expression of the generalized second law of thermodynamics. On the other hand, given that the entropy bounds involve inverse powers of $\hbar G$ it is very possible that they are deep clues to the structure of the fundamental theory, and that some version of the holographic principle may even turn out to be a fundamental principle of the quantum theory of gravity. If so it will be the first principle that is genuinely quantum gravitational, rather than just being imported from general relativity or quantum theory.

But if this is to be the case the true, fundamental statement of the holographic principle must be made in the language of some background independent quantum theory of cosmology. This is likely to be phrased in a different language than its semiclassical formulation, for the same reason that the laws of thermodynamics are expressed in very different language when expressed fundamentally in quantum statistical mechanics than they are when one first meets them as characterizations of the thermodynamic limit. The problem is then to discover what features of the entropy bounds and holographic principles so far discussed might be artifacts of the semiclassical limit, and to separate these from the principle's true content.

In this paper a line of reasoning is presented, which leads to the identification of a form of the holographic principle that can survive passage to a background independent quantum theory of cosmology. This is called the weak holographic principle[10]; it is both logically weaker and conceptually more radical than the forms of the principle originally contemplated in the

literature. It is logically weaker in that it makes no assertion as to a relationship between a bulk and a boundary theory. As has been found also by Fischler and Susskind[6] and others[7, 8, 9], that idea already fails at the semiclassical level for cosmological theories. We found in our investigations further reasons why such a strong form of the principle cannot be fundamental. Instead, the weak holographic principle comes into a background independent quantum theory of cosmology as a framework for that theory's interpretation and measurement theory. Its role is to constrain the quantum causal structure of a quantum spacetime in a way that connects the geometry of the surfaces on which measurements may be made with a measure of the information that those measurements may produce. In this context the entropy bound becomes a definition, by which the notion of geometry is reduced to more fundamental notions coming from the quantum theory of cosmology. To put it simply, the Bekenstein bound is turned on its head and the notion of area is reduced fundamentally to a measure of the flow of quantum information. This form of the principle was first suggested in [10]; the present paper can be taken as an argument that no form of the principle which is logically stronger, or conceptually less radical, can survive passage to a background independent quantum theory of gravity.

One difficulty of the subject is that different authors have proposed different ideas under the name of the holographic hypothesis or principle. It is necessary first to bring a bit of order to the situation by classifying the different proposals in a way that uses a common language and makes clear their logical relations to each other. To do this we use the language of *screens.* In this paper a screen will always mean an instantaneous, spacelike, two dimensional surface[1] on which quantum mechanical measurements are made. These will always be measurements of fields on the surface, which then result in information concerning the causal past of the surface.

To make progress it is first of all necessary to distinguish between *entropy bounds* and holographic *principles.* The former are limitations on the degrees of freedom attributable to either the screens themselves, or spacelike or null surfaces bounded by the screens. In the literature entropy bounds are sometimes called holographic bounds, but we will stick to the former expression to avoid confusion. A holographic principle extends an entropy bound by postulating a form of dynamics in which the quantum evolution of the spacetime and matter fields is described in terms of observables measurable on the screens.

[1]Or, in $D+1$ dimensions a surface of dimension $D-1$.

We find that the different entropy bounds and holographic principles that have been proposed fall each into three classes, which we call the "strong" "null" and "weak" forms[10]. The different entropy bounds all postulate that some measure of information or of a"number of degrees of freedom" is bounded by the area of a screen. The strong forms are those that postulate that the bound applies to the degrees of freedom on a spacelike surface bounded by the screen. The null forms are those, suggested by Fischler and Susskind[6] and put in a very elegant form by Bousso[8, 9] and Flanagan, Marolf and Wald[11], in which there is a bound on the number of degrees of freedom of certain null surfaces, bounded by the screens. The weak form, proposed with Markopoulou in [10], postulates only a relationship between the area of the screens and the dimension of the Hilbert spaces which provide representations of algebras of observables on them.

The main conclusion of this paper will be that the strong entropy bound cannot hold in a cosmological theory, and that the null form may only hold in a semiclassical theory in which quantized matter degrees of freedom evolve on a fixed spacetime manifold, but cannot survive the quantization of the gravitational field. It appears that only the weak form, which as the name suggests is logically weaker and therefore requires less, may survive in a full theory of quantum gravity.

The different forms of the entropy bound stem from different interpretations that may be given to the Bekenstein bound[4]. These, may be called the strong and weak Bekenstein bounds. They are presented in the next section. The distinction is that the weak form bounds only the information measurable by observers just outside the horizon of a black hole, while the strong form bounds the total number of degrees of freedom measurable in the interior of the horizon. We find that only the weak form is required by the usual arguments based on the laws of thermodynamics. The strong form of the Bekenstein bound follows only if we add an independent assumption, which is that the number of degrees of freedom measurable on the interior does not exceed those measurable on the exterior. We call this the *strong entropy assumption.* It must be postulated independently, as it does not follow from any argument which involves only measurements made exterior to the black hole horizon. This conclusion has been reached also by Jacobson[12]. One of the conclusions of this paper is that the strong entropy assumption is false. Among other things it is inconsistent with both inflation and gravitational collapse.

The strong, null and weak forms of the holographic principle depend on the corresponding forms of the entropy bounds. They extend each of them

by giving a framework for dynamics. As only the weak form of the entropy bound seems to be possible in a full quantum theory of gravity, only a weak form of the holographic principle may be true in such a theory.

The author is aware that this is not a completely welcome conclusion. In fact, it goes against his own proposal for a bulk to boundary isomorphism in quantum general relativity and supergravity[13, 14, 15]. It unfortunately conflicts also with some of the hopes which have been held concerning the AdS/CFT correspondence[16, 17]. It is then necessary to discover if there is any conflict between the conclusions reached here and the many results which have been found which support some version of the AdS/CFT conjecture[17]. We find that there is not. This is likely because most of the results so far found are consequences of much weaker assumptions, which involve only the transformation properties of observables under the supersymmetric extension of $SO(D, 2)$. In fact, Rehren[18] has shown rigorously that a correspondence will always exist between theories on an AdS_D background and conformal field theories on $Mink_{D-1}$, subject only to the condition that the latter exist. The results so far found concerning AdS/CFT then may hold as a consequence of this theorem. To the extent that this is true they do not then provide any independent evidence for a strong version of the holographic principle that would go beyond this case.

Does this mean that there is something wrong with the idea that the holographic principle may play a role in string theory, as was suggested by the original arguments for the AdS/CFT correspondence[16, 17]? Certainly, not, what it means is that if it is to go beyond the level of description in terms of dynamics of strings and branes in fixed classical background spacetimes, string theory must be formulated in a background independent langauge. Forms of the holographic principle that may suffice in the context of physics on a single fixed background are likely to be of limited validity, but the results of our arguments is that there are forms of the bounds and principle that may hold in a background independent theory. In fact, as we argued in [10], the weak holographic principle may hold in background independent formulations of string theory[54].

We now give an outline of the paper, emphasizing the logical structure of its argument.

This paper is divided into two parts. The first concerns entropy bounds. In the next section we discuss the weak and strong versions of the Bekenstein bound and establish the claims made above.

In section 3 we turn our attention to cosmology and find that the weak and strong Bekenstein bounds each imply a cosmological bound, called re-

spectively the weak and strong cosmological entropy bounds. As in the non-cosmological case, the strong form cannot be derived without making the strong entropy assumption.

In section 4 we then give five counterexamples to the strong cosmological holographic bound. These are

1. The gravitational collapse problem.
2. The inflation problem.
3. The wiggly surface problem.
4. The two-sided problem.
5. The throat problem.

The conclusion is that the strong cosmological entropy bound is false. Since this followed from known physics plus the strong entropy assumption, the likely conclusion is that the latter cannot hold in a gravitational theory.

We then describe, in section 5, the new cosmological entropy bound proposed by Bousso[8], which we call the null entropy bound. It seems to be correct at the classical and semiclassical level, as a bound on the matter entropy in a fixed spacetime. However, to play a role in a quantum theory of gravity, an entropy bound should extend to a case in which the gravitational degrees of freedom are dynamical. In section 6 we present two arguments why the null entropy cannot hold once the gravitational degrees of freedom are turned on, either classically or quantum mechanically.

The only form of an entropy bound that can then survive at the level of a full quantum theory of cosmology is then the weak form. This is the conclusion of our discussion of entropy bounds.

The second part of the paper concerns the question of whether, given the conclusions reached in the first part, there is any form of a holographic principle that may hold in a quantum theory of gravity. Such a principle must give a framework within which to describe the dynamics of the degrees of freedom constrained by the entropy bounds. Most forms of the holographic principle which have been discussed assume the strong form of the entropy bound. Dynamics is then formulated in terms of a map between the bulk and boundary Hilbert spaces that preserves unitary evolution. Since the strong form of the entropy bound seems to disagree with things we believe to be true, such a strong form of the holographic principle is ruled out, at least for the case of gravitational theories.

In section 8 we consider this situation carefully, as it is not what many people's intuition seems to suggest. We show that there is no contradiction with what we know, even taking into account all the results found concerning the AdS/CFT correspondence. We also note that an elegant solution to the black hole information paradox is still available.

We then raise the question of whether there might be a weaker form of the holographic principle which may still hold. We consider first, in section 9, the question of whether some form of the holographic principle may be associated with the null entropy bound. We reach the conclusion that such a principle may exist, but it must be based on a modification of quantum theory in which there are many Hilbert spaces, one for each screen.

However, if the null entropy bound cannot survive the turning on of the gravitational degrees of freedom, neither can the null version of the holographic principle. We are then left with the question of whether there might be a weak version of the holographic principle, which would correspond with the weak entropy bound. We first, in section 10, discuss the question of which two surfaces may be screens in such a formulation. We come to the conclusion that none of the possible criteria for distinguishing screens from other two surfaces can survive passage to the full quantum theory. Therefor every spacelike two surface may be considered a screen. This opens up the possibility of defining geometry in terms of the properties of screens, rather than visa versa.

In section 11 we then list the conclusions of the argument reached till this point, which then may be considered to motivate and constrain the possible forms of a weak holographic principle. One possible form of the principle, given in [10] is then reviewed in section 12. There we also describe briefly two independent arguments for a weak form of the holographic principle. These come from considerations of the role of quasi-local observables in general relativity and, in a form made originally by Crane[3], from relational formulations of quantum cosmology.

The paper then closes with a short summary of the main conclusions.

Part I
ENTROPY BOUNDS

2 The weak and strong forms of the Bekenstein bound

The different possible entropy bounds, as well as the different possible forms that the holographic principle might take, have their origin in the fact that different meanings may be given to the entropy of a black hole. To see this, let us distinguish

- **The thermodynamic black hole entropy**

$$S_{bh} = \frac{A}{4G\hbar} \tag{1}$$

  which enters the laws of black hole mechanics.(here A is the area of the black hole horizon.)

- I_{bh}^{weak}**, Weak black hole entropy** This is a measure of how much information an observer external to its horizon can gain about its interior, from measurements made outside the horizon. Besides the mass, angular momentum and charges, this includes measurements of the quanta emitted by the black hole.

- I_{bh}^{strong}**, Strong black hole entropy** This is a measure of how much information is contained in the interior of the black hole. This can also be expressed as the "number of degrees of freedom" inside the black hole, or the number of distinct ways in which it might have been assembled.

We may note that the generalized second law requires require that

$$I_{bh}^{weak} \leq S_{bh} \tag{2}$$

This is because all of the arguments for it concern exchanges of matter and radiation between the black hole and observers situated outside its horizon. They do so because they assume that the semiclassical approximation is valid so that the only way that matter or information can cross from the interior to the exterior is in the form of thermalized Hawking radiation.

We may note that the number of quanta emitted by a black hole during Hawking radiation is on the order of (1), this is consistent with eq. (2).

We are, of course, ignorant of what happens when a black hole evaporates to any state in which it has a mass of the order of the Planck scale. The only

thing we know with any confidence is that the semiclassical approximation breaks down. To attack this problem many authors either implicitly or explicitly make the following assumption

- **Strong entropy assumption:**

$$I_{bh}^{strong} = I_{bh}^{weak} \tag{3}$$

This is an attractive assumption. For example, it suggests that there may be a single Hilbert space, which is a representation of operators at infinity, within which the full evolution of a system, from prior to black hole collapse to the aftermath of complete evaporation, may be represented as unitary evolution. However, we should note that the logic is not symmetric, as there are remnant scenarios under which unitary evolution does not imply (3). Nor can (3) be supported by any arguments for the generalized second law, as they concern only exchanges of material across the black hole horizon described in the semiclassical approximation. Thus, it is logically possible that (3) is false and that

$$I_{bh}^{strong} > I_{bh}^{weak} \tag{4}$$

It is also possible that I_{bh}^{strong} is not even a well defined quantity.

The arguments which are usually taken as supporting some version of the holographic hypothesis depend strongly on whether or not (3) is assumed. To see this, let us run the standard Bekenstein argument[2].

The Bekenstein argument

Consider a timelike three dimensional region $\mathcal{R}$ of an asymptotically flat spacetime $\mathcal{M}$, the quantum dynamics of which we wish to study. We will assume $\mathcal{R}$ has topology $\mathcal{R} = \Sigma \times R$, where Σ is a spatial manifold. We will restrict attention to the physics within $\mathcal{R}$ by the imposition of boundary conditions on $\partial\mathcal{R} = \partial\Sigma \times R$. We will denote $\mathcal{S} = \partial\Sigma$. These will restrict the degrees of freedom of the gravitational field on the boundary; as a result a reduced set of observables will be able to vary at the boundary.

Let $\mathcal{A}_{\mathcal{S}}$ be the complete algebra of the unconstrained observables on a spatial slice of the boundary, $\mathcal{S}$. This will have a representation on a

[2]This form of the argument is taken from [19], but it is due originally to Bekenstein[4]. There is also some confusion because a different bound, $S < RE$, where E is the energy of a system was also postulated by Bekenstein to hold in ordinary quantum field theory, in the absence of gravity. The bound we need,(2) is logically weaker than that, and so arguments against this hypothesis do not necessarily contradict (2).

Hilbert space $\mathcal{H}_{\mathcal{S}}$. We will always assume that $\mathcal{H}_{\mathcal{S}}$ is the smallest non-trivial representation, i.e. it contains no operators that commute with the representatives of $\mathcal{A}_{\mathcal{S}}$. We will call these the boundary observables algebra and boundary Hilbert space. We may assume that among the elements of $\mathcal{A}_{\mathcal{S}}$ are the Hamiltonian, $H_{\mathcal{S}}$ and the areas of regions $\mathcal{I}$ of the boundary $\mathcal{S}$, which I will denote $A[\mathcal{I}]$. Recall that in general relativity the Hamiltonian is, up to terms proportional to constraints, defined as an integral on the boundary and is thus an element of $\mathcal{A}_{\mathcal{S}}$.

Since the system contains gravitation, we may assume that among the spectrum of states in $\mathcal{H}_{\mathcal{S}}$ are a subspace which correspond to the presence of black holes in Σ. These are semiclassical statistical states, and we will assume that their statistical entropies, given by the dimensions of the corresponding subspaces of $\mathcal{H}_{\mathcal{S}}$ are given by the usual formula (1), in the semiclassical limit when their masses and areas are large in Planck units.

We will consider only systems in thermal equilibrium. This rules out examples from cosmology or astrophysics in which the thermalization time or light-crossing time is longer than the time in which the system will gravitationally collapse.

The argument is simplest in the case that we assume that the induced metric on $\mathcal{S}$ is spherical, up to small perturbations corresponding to weak gravitational waves passing through the boundary. The argument proceeds by contradiction. We assume that the region Σ can contain an object $\mathcal{O}$ *whose complete specification in the boundary Hilbert space* H_{Σ} requires an amount of information $I_{\mathcal{O}}$ which is larger than

$$I_{\mathcal{S}} = \frac{A[\mathcal{S}]}{4 l_{Pl}^2} \tag{5}$$

which is of course the entropy of a black hole whose horizon just fits inside of $\mathcal{S}$.

Let us assume that initially we know nothing about $\mathcal{O}$, so that $I_{\mathcal{O}}$ is a measure of the entropy of the system. However, with no other information we can conclude that $\mathcal{O}$ is not a black hole, as the largest entropy that could be contained in any black hole in Σ is $I_{\mathcal{S}}$. We may then argue, using the Hoop theorem[20] that the energy contained within Σ (as measured either by a quasi-local energy on the surface or at infinity) must be less than that in a black hole whose horizon has area $\mathcal{A}[\mathcal{S}]$. But this being the case we can now add energy slowly to the system to bring it up through an adiabatic transformation to the mass of that black hole. By the hoop theorem this

will have the result of transforming $\mathcal{O}$ into the black hole whose horizon just fits inside the sphere $\mathcal{S}$.

This can be done by dropping quanta slowly into the black hole, in a way that does not raise the entropy of its exterior. As a result, once the black hole has formed we know the entropy of the system, it is $I_{\mathcal{S}}$. But we started with a system with entropy $I_{\mathcal{O}}$, which we assumed is larger. Thus, we have violated the generalized second law of thermodynamics. The only way to avoid this is if $I_{\mathcal{O}} < I_{\mathcal{S}}$.

Since this is a bound on the total information that could be represented in $\mathcal{H}_{\mathcal{S}}$, we have

$$\ln Dim\,[\mathcal{H}_{\mathcal{S}}] = I_{\mathcal{S}} = \frac{A[\mathcal{S}]}{4l_{Pl}^2} \tag{6}$$

We may remark that this argument employs a mixture of classical, statistical and semiclassical reasoning. For example, it assumes both that the hoop theorem from classical general relativity applies, in the case of black hole masses large in Planck units, to real, quantum black holes. One might attempt to make a detailed argument that this must be the case if the quantum theory is to have a good classical limit. However worthy of a task, this will not be pursued here, as it is unlikely that any such argument can be elevated to establish the necessity, rather than plausibility of the Bekenstein bound, in the absence of a complete theory of quantum gravity.

What does the Bekenstein argument imply?

We may note that the above argument involves only the weak form of the black hole entropy, I_{bh}^{weak}. This is because what is under discussion is the description of the system *as given by states in the boundary Hilbert space* $\mathcal{H}_{\mathcal{S}}$. This is, as emphasized, a representation of the algebra of operators $\mathcal{A}_{\mathcal{S}}$ *measurable on the boundary.* This is sufficient to make the argument, as the crucial steps involves a) use of the hoop theorem and b) adiabatically feeding energy into the system, both of which concern only measurements or operations which may be made on the boundary. The conclusion of the argument then only concerns the dimension of $\mathcal{H}_{\mathcal{S}}$, and hence only external measurements. We may express this by saying that the argument demonstrates the

- **Weak Bekenstein bound** Let a system Σ be defined by the identification of a fixed boundary $\partial\Sigma = \mathcal{S}$, and a Hilbert space $\mathcal{H}_{\mathcal{S}}$ be defined as the smallest faithful representation of the algebra of observables $\mathcal{A}_{\mathcal{S}}$

measurable on the boundary only. Either the area $A[\mathcal{S}]$, is fixed or is in $\mathcal{A}_\mathcal{S}$. In the first case,

$$Dim\mathcal{H}_\mathcal{S} \ \le \ e^{\frac{A[\mathcal{S}]}{4G\hbar}} \tag{7}$$

where G is the *physical, macroscopic* Newton's constant. In the case that $A[\mathcal{S}] \in \mathcal{A}_\mathcal{S}$, the Hilbert space $\mathcal{H}_\mathcal{S}$ must be decomposable into eigenspaces of $A[\mathcal{S}]$ such that (7) is true in each.

Without further assumptions this implies nothing for quantities that refer essentially to the "bulk" such as "the number of degrees of freedom contained in the region Σ". In order that the argument goes further we may add to it the independent assumption that (3) holds. This then does imply the

- **Strong Bekenstein bound.** Under the same assumptions, let $\mathcal{H}_{bulk}$ be the smallest faithful representation of the algebra of local observables measurable in the interior of Σ. Then

$$Dim\mathcal{H}_{bulk} \ \le \ e^{\frac{A[\mathcal{S}]}{4G\hbar}} \tag{8}$$

To summarize, the important points are that the generalized second law implies only the weak Bekenstein bound, and that the strong entropy assumption is an independent hypothesis. The logic is then that

$$\text{Black hole thermodynamics+second law+Hoop theorem} \rightarrow \text{weak Bekenstein bound.} \tag{9}$$

and

$$\text{weak Bekenstein bound+strong entropy assumption} \rightarrow \text{strong Bekenstein bound} \tag{10}$$

3 The weak and strong cosmological entropy bounds

We now turn to the question of whether some form of a holographic bound may apply to a cosmological theory in which no boundary conditions have been enforced. Let us consider any closed surface, $\mathcal{S}$, which bounds a region $\mathcal{R}$ in a compact spatial slice, Σ of a cosmological spacetime. No boundary conditions have been imposed on $\mathcal{S}$, thus its interior, $\mathcal{R}$ should contain more "degrees of freedom" than would be the case were boundary conditions

imposed, because boundary conditions always act by suppressing degrees of freedom, and hence reducing the number of classical solutions, in the neighborhood of the boundary. This means that the above bounds have implications for the representation spaces of algebras of observables that describe regions without boundary conditions imposed.

To make this precise, let $\mathcal{A}_{\mathcal{S}}^{free}$ be the total algebra of observables measurable on $\mathcal{S}$, when no boundary conditions have been imposed, and let $\mathcal{A}_{\mathcal{S}}^{bc}$ be the algebra of observables which remain unconstrained when a particular set of boundary conditions have been imposed. Let $\mathcal{H}_{\mathcal{S}}^{free}$ and $\mathcal{H}_{\mathcal{S}}^{bc}$ be their corresponding representation spaces. Clearly $\mathcal{A}_{\mathcal{S}}^{bc} \subset \mathcal{A}_{\mathcal{S}}^{free}$, which implies that

$$\mathcal{H}_{\mathcal{S}}^{bc} \subset \mathcal{H}_{\mathcal{S}}^{free} \tag{11}$$

This means that

$$dim(\mathcal{H}_{\mathcal{S}}^{bc}) \leq dim(\mathcal{H}_{\mathcal{S}}^{free}) \tag{12}$$

We assume that the set of variables which are fixed by the boundary conditions make up a commuting subalgebra of $\mathcal{A}_{\mathcal{S}}^{free}$, otherwise they could not all be imposed at once. It is also natural to assume that the amount of information concerning the state in $\mathcal{H}_{\mathcal{S}}^{free}$ which is necessary to fix the boundary conditions is proportional to the area $A[\mathcal{S}]$. It then follows that

$$\ln dim(\mathcal{H}_{\mathcal{S}}^{free}) = \ln dim(\mathcal{H}_{\mathcal{S}}^{bc}) + \alpha \frac{A[\mathcal{S}]}{G\hbar} \tag{13}$$

where α is some dimensionless constant. We call this the *boundary condition area assumption.*

By putting this together with the weak Bekenstein bound for the system with boundary conditions, (7) we find that,

$$\ln dim(\mathcal{H}_{\mathcal{S}}^{free}) = \left(\frac{1}{4} + \alpha \right) \frac{A[\mathcal{S}]}{G\hbar} \tag{14}$$

Note that this follows that even though no boundary conditions have been applied at $\mathcal{S}$. Thus, we have a bound that applies to surfaces inside cosmological spacetimes.

- **Weak cosmological entropy bound.** Let $\mathcal{S}$ be a spacelike surface of spacetime codimension 2 that splits a complete spacelike hypersurface into two regions, let $\mathcal{A}_{\mathcal{S}}^{free}$ be the complete algebra of observables

measurable on $\mathcal{S}$ and let $\mathcal{H}_{\mathcal{S}}^{free}$ be its smallest representation space. Then,

$$\ln dim(\mathcal{H}_{\mathcal{S}}^{free}) = C\frac{A[\mathcal{S}]}{G\hbar} \tag{15}$$

for some $\mathcal{S}$ independent constant C.

There is also a strong form of this argument. If we assume the strong entropy assumption, (3), then the same argument leads to

- **Strong cosmological entropy bound.** Let $\mathcal{S}$ be a spacelike surface of spacetime codimension 2 that splits a complete spacelike hypersurface into two regions, let $\mathcal{A}_{\mathcal{S}}^{strong}$ be the complete algebra of observables measurable on the interior of $\mathcal{S}$ and let $\mathcal{H}_{\mathcal{S}}^{strong}$ be its smallest representation space. Then,

$$\ln dim(\mathcal{H}_{\mathcal{S}}^{strong}) = C\frac{A[\mathcal{S}]}{G\hbar}. \tag{16}$$

We again summarize the logic,

$$\text{weak bekenstein bound+bc. area assumption} \rightarrow \text{weak cosmological entropy bound} \tag{17}$$

$$\begin{aligned}\text{weak cosmological entropy bound} \quad + \quad & \text{srong entropy assumption} \\ & \rightarrow \text{strong cosmological entropy bound}\end{aligned} \tag{18}$$

4 Counterexamples to the strong cosmological entropy bound

Unfortunately, the strong cosmological entropy bound contradicts known physics. This is shown by the following five counterexamples.

The gravitational collapse problem

Consider a co-moving region $R(\tau)$ in a closed Friedman Robertson cosmology, where τ is the standard FRW time coordinate. Let us assume that at the time of maximum expansion, τ_0, $R(\tau_0)$ contains a uniform gas with entropy $S(\tau_0)$, while its boundary has area $A(\tau_0)$. If we assume the strong cosmological holographic bound (16) then $S_0 < A(\tau_0)/4G\hbar$. However as

the volume of the universes decreases after τ_0, so will $A(\tau)$. But, by the second law $S(\tau)$ will increase. There will then be a time τ_1 such that $S_0 = A(\tau_0)/4G\hbar$. After that the strong cosmological bound will be violated. Since the spacetime geometry, and the distribution of gas, are uniform, the bound cannot be saved by the formation of a black hole. A similar problem occurs for boxes of radiation dropped into black holes.

Note that this example escapes the conditions of the Bekenstein argument because the universe is not asymptotically flat.

The inflation problem

It is not hard to see that inflation provides counterexamples to the strong cosmological holographic bound, arising from the fact that in the aftermath of inflation a universe will have approximately uniform regions exponentially larger than the Hubble scale[3]. The real horizon size R_H at any given time can then be arbitrarily large compared to the hubble scale H, and still contain entropy created in a single causally connected region since the initial singularity.

To see this we follow the exposition of Kolb and Turner, [21]. We follow a causally connected region which begins as a patch of the size of H^{-1} at the time inflation starts, which is equal to

$$H^{-1} = \frac{m_P}{M^2} = R_{initial} \tag{19}$$

where M is a mass scale associated with the inflaton potential, which is between the Planck scale and the weak scale. The past lightcone of this patch will just touch[4] the initial surface $t = 0$.

There is then a period of inflation, in which the patch expands to a size $e^N R_{initial}$ which is followed by a period of reheating, during which it expands by a further factor of

$$\left(\frac{M^4}{T^4}\right)^{1/3} \tag{20}$$

where T is the reheating temperature. During reheating a bath of black body radiation is created from the dissipation of the inflaton field with temperature T, after which the inflation sits in the bottom of its potential

[3] This argument has been raised independently in [7].

[4] so that it corresponds to the case Fischler and Susskind[6] considered.

and the universe is, to a very good approximation, spatially flat. Agreement with observations seems to require

$$N > 60 \tag{21}$$

Just after reheating the region we may try to apply the Bekenstein bound to the huge bubble that the patch has grown up to be, which is of radius

$$R_r = l_P e^N (\frac{m_p}{T})^{4/3} (\frac{m_p}{M})^{2/3} \tag{22}$$

as space is flat it encloses a volume $8\pi/3R_r^3$, which contains an entropy

$$S_r = \frac{8\pi\nu}{3} T^3 R_r^3 = \frac{8\pi\nu}{3} e^{3N} \frac{m_P^3}{M^2 T} \tag{23}$$

where

$$\nu = \frac{\pi^2}{40} g^* \tag{24}$$

is of order 20 as g^* is of order 100. If we ask that this entropy be bounded by 1/4 the horizon area in Planck units we have

$$S_r \leq 4\pi^2 R_r^2 \tag{25}$$

This seems to put a bound on N which rules out inflation. This happens because the entropy contained in the horizon grows as e^{3N} while its area only grows as e^{2N}. The result is that (6) implies a strict bound on R_r,

$$R_r \leq \frac{3}{2\nu} (\frac{m_P}{T})^3 \tag{26}$$

which means that the number of efoldings is bounded by

$$N \leq \ln \frac{3}{2\nu} + \frac{5}{3} \ln \frac{m_P}{T} - \frac{2}{3} \ln \frac{m_P}{M}. \tag{27}$$

If we use physically reasonable values for M and T it is impossible that there were as many as 60 e-foldings. Thus, the strong cosmological holographic bound (16) is in conflict with the standard inflationary scenario.

What went wrong? To see that the problem is inflation we may note that if we were ignorant of inflation having taken place, and took the inverse hubble scale H^{-1} for the horizon size just after reheating instead of the

much larger R_r the strong cosmological holographic bound (16) yields the reasonable statement that,

$$T \leq \sqrt{\frac{2}{3\pi\nu}} m_P. \tag{28}$$

So, we may wonder, why isn't the huge region of radius R_r unstable to gravitational collapse. It is clearly, for it has a Schwarzchild radius

$$R_{Sch} = l_P 4\pi\nu e^{3N} (\frac{m_P}{M})^2 \tag{29}$$

Requiring that $R_r > R_{Sch}$ yields an even stricter bound on the number of efoldings,

$$N < \frac{2}{3}\ln(\frac{M}{T}) - \frac{1}{2}\ln(4\pi\nu) \tag{30}$$

So the region created by inflation is unstable to gravitational collapse. Given any inhomogeneities these will grow and, if they are large enough, form black holes. But this is nothing new, it is just the process of galaxy and structure formation. Indeed, because Ω is now very close to one, large regions of the bubble must be unstable to the gravitational collapse that must eventually occur in any region in which, locally, $\Omega > 1$.

All of the entropy contained in the region blown up by inflation corresponds to ordinary thermal fluctuations in the radiation produced by reheating. As the process of reheating is an ordinary physical process, and as the inflation field may be assumed to have been in a coherent state before inflation began, we must believe that there are all the degrees of freedom in that region given naively by the entropy we have computed. That entropy is, indeed, a measure of how much information would be needed to determine the precise quantum state which resulted from the process of reheating.

However, because the causal horizon has blown up to such a big size from inflation, that information required is much much greater, for standard inflation models, than the area of the horizon just after reheating in Planck units. (recall indeed that in most standard inflationary scenarios, N is much great than its minimal value of 60 and may easily be $> 10^4$.

The inflation problem shows that there can be a surface in the universe, the causal horizon, S_{ch}, whose information content, proportional to its area, is too small to reconstruct the state of all the thermal photons in its interior. Is this a problem for a consistent cosmological formulation of the holographic principle? To answer this we have to ask what information about the interior may arrive at a surface at the causal horizon. The key point is that because

of the exponential expansion, an observer there is not able to observe that thermalization has taken place over all but a small shell of the interior, in the neighborhood of S_{ch}. For the rest the observer can see causal effects only from the region prior to inflation and reheating, when a description in terms of a pure state is completely adequate. This is because, prior to reheating, the state during that era is very close to the vacuum, and hence can be described with very little information.

We may contrast this with the information available for a surface S_{oh} within the conventional ordinary horizon, with $r < H$, the Hubble scale. An observer at such a surface sees the region in the interior after reheating and thus sees a thermal distribution of photons. But the region is small enough that enough information is available on S_{ohs} to reconstruct the states of those thermal photons.

Is there a conflict between these two descriptions? No, not if one takes into account two facts. First, the observer at the smaller surface must see a mixed state, because the photons in the interior of S_{oh} will be correlated with photons in their exterior. Only from the much larger surface S_{ch} can an observer reconstruct a pure state, because they see all the correlations between the thermal photons created by the inflation and subsequent reheating. However, it takes much *less* information to describe the pure state than to describe the thermal state, in the whole of the interior of S_{ch} because once the quantum correlations are neglected one must account for all the individual states of all the individual photons.

Second, because of causality, the observer at the larger surface S_{ch} is only able to observe the state in the interior at a much earlier time, before inflation and reheating, when a pure state description, requiring much less information, is appropriate.

This examples teaches us that the holographic bound concerns only the information available on a surface S_{ch} by virtue of quanta which reach it from the interior. This *is not the same information* as would be required to reconstruct the state of the system on a spacelike surface spanning S_{oh}. They are different because by causality, the information available on a surface is that information that can reach the surface by causal propagation of information from the interior[5].

[5]One may ask why this example does not provide a counterexample to the Bekenstein bound. The reason is that a large region of an inflationary universe is excluded because the light crossing, and hence thermalization time is long compared to the time scale for subregions to gravitationally collapse. These cases were excluded explicitly in the argument, because the step in which one evolves equilibrium states adiabatically by slowly

The wiggly surface problem

Next, we consider three two-dimensional surfaces in a compact spatial slice Σ. The first two are $\mathcal{S}_1$ and $\mathcal{S}_2$, with $\mathcal{S}_1 \in Int(\mathcal{S}_2)$, where $Int(\mathcal{S}_2)$ is the region in Σ to the interior of $\mathcal{S}_2$. For example, these could be constant r surfaces in a constant t slice of Schwarzchild-De Sitter, (using standard coordinates) with $r_1 < r_2$. In such a case we can choose $A_1 < A_2$, which implies that less information could be represented on A_1 than on A_2. This makes sense because there is a region between the two surfaces that contains physics that may be observed by the observer at S_2 that is not observed by the observer at S_1.

Now consider a surface S_1', just to the interior of A_1 which is gotten by displacing S_1 slightly into its interior, and then wiggling it, for example by superposing on it some set of waves. The wiggled surface can easily have area $A_1' > A_2 > A_1$. What are we to make of the apparent fact that the surface S_1', can contain an amount of information greater than the other two? If $\mathcal{S}_1$ contains all the information about its interior, then the information coded on S_1' cannot be greater than that coded on S_1. But as it has a greater area, it seems to have a greater information capacity.

The wiggly surface problem tells us that the area that is relevant for the measure of information is not the actual area of the surface $\mathcal{S}$. Rather it must correspond to the information reaching $\mathcal{S}$ from its interior. This can be achieved if we identify the surface $\mathcal{S}$ with a cross-section $\sigma(\mathcal{S})$ of a congruence of light rays which intersect $\mathcal{S}$. We may note that the original arguments of Susskind and others were phrased in terms of such congruences of light rays[2, 22].

This has an important implication. The bounds on the information on a screen, $\mathcal{S}$, cannot refer just to that surface. It must refer instead to the minimal area of cross-sections through a congruence of light rays that arrive at S from the past.

The two-sided problem

Consider now a surface $\mathcal{S}$ of area A and topology S^2 embedded in a compact spatial manifold Σ, which we take to be an S^3. Then $\mathcal{S}$ splits the universe into two three-balls $B^{\pm}$,such that $\Sigma = B^+ \cup B^-$, each bounded by a side of $\mathcal{S}$, which we will call $\mathcal{S}^{\pm}$. The problem is that if $\mathcal{S}$ is a screen there are actually two possible holographic descriptions, associated with $\mathcal{S}^{\pm}$. One

dripping in energy cannot be realized.

should codes a description of B^+, the other should codes a description of B^-.

Assume that the universe is in a semiclassical state, so that we may to a reasonable approximation describe the geometry of Σ classically. Then consider taking A in Planck units smaller and smaller. The holographic principle must associate to each screen $\mathcal{S}^\pm$ a state space $\mathcal{H}^\pm$. These must have the same dimension, which is shrinking as $e^{A/4l_{pl}^2}$. But one of the two balls, say B^+ contains almost the whole universe, while the other B^- contains only a small region. Since the universe is classical, it is large in Plank units. We have no problem imagining that the physics in B^- is coded in a state space of dimension bounded by $e^{A/4l_{pl}^2}$, as that is a very small region. But it seems the physics in B^+, which is almost the entire universe must also be describable in terms of state space of this small dimension.

This seems at first paradoxical. It seems that this will require that an arbitrarily small screen may be required to code information about an arbitrarily large region. Is it possible to resolve this paradox?

It is, if we apply the conclusion of the wiggly surface problem. We see that what is relevant is not the information that may reside on a spacelike surfaces spanning $\mathcal{S}^+$ and $\mathcal{S}^-$ but the information reaching those surfaces transmitted by a congruence of light rays from their pasts. This is not necessarily the same thing because to transmit information the light must be focused on the surface. We must then consider the cost in entropy of focusing light from a large universe onto a small surface. The apparent loss of information in recording the holographic image of the large universe on a small surface may be explained if the entropy generated (or information required) by the processes of focusing the light on the surface is large.

Consider a small surface, S, with area of $100 l_{pl}^2$ in a large universe with volume $10^{180} l_{Pl}^3$. In order for information about the state of the whole universe to arrive at S a congruence of light rays originating all over the universe must be focused very precisely so that its focal plane is the surface S at a fixed time t (measured by a clock at S.) In a universe in thermal equilibrium, the operation of focusing a congruence of light rays so precisely, in a manner that compensates for all the structure in the gravitational field due to the presence and motion of matter will generate a huge amount of entropy. The result[6] is that a great deal of information about the universe

[6]This can be made more quantitative, and will be elsewhere. Note that as we are discussion the properties of a full quantum theory of gravity, counter-examples based on classical solutions with isometries are irrelevant. It is always possible to find counterex-

is then stored, not on S, but in the configuration of matter or lenses that had to be organized in order to get the light to focus on S.

This will only be unnecessary if the universe is completely symmetric. However such a universe will, by virtue of its symmetry, contain only a few bits of information[7].

The throat problem

There are spacetimes, (M, g), in which the following situation occurs. (M, g) contains a spacelike slices Σ, with three embedded two dimensional surfaces $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$, with $\mathcal{S}_3 \in Int(\mathcal{S}_2) \in Int(\mathcal{S}_1)$, but in which their areas satisfy $A_1 > A_2 < A_3$. This can happen if, for example, (M, g) is a Kruskal completion of a black hole solution, $\mathcal{S}_1$ is a surface outside the horizon, $\mathcal{S}_2$ is the throat of the black hole, and $\mathcal{S}_3$ is a two surface which is at smaller r than the throat, and is topologically contained in it, but yet has larger area[8].

There is a large class of such examples in which the Kruskal spacetime is truncated inside the throat, and a compact region is glued on containing matter, describing what is sometimes called a "baby universe". Such universes are conjectured to arise in a large class of scenarios in which quantum effects lead to an avoidance of the formation of the singularity. The problem is that the baby universe is topologically inside $\mathcal{S}_2$, but contains 2-surfaces which have a larger area.

To make the problem more worrying we can also imagine that $A_1 > A_3$. In such situations it seems like an observer inside the baby universe at $\mathcal{S}_1$ can have more information about the contents of the baby universe than can the observer at $\mathcal{S}_3$ who is outside the horizon of the black hole. This means that the observer at $\mathcal{S}_3$ may not have enough information to reconstruct the whole state in the interior of the black hole.

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amples to statistical theorems from examples with non-generic symmetries, consider for example the ellipsoid with a point source of light at one focal point. It apparently will not stay in equilibrium.

[7] It might be objected that there is a limit in which the area, and hence the information capacity, of the surface is strictly zero. But this is not true, under quite generic assumptions in quantum gravity and supergravity there is a minimal unit of area, which is greater than zero[23, 24, 25].

[8] This example is not cosmological, but can easily be made so by inserting the black hole into a cosmological solution.

5 Identifying the wrong assumption

It is difficult to see how to escape the conclusion that the strong cosmological entropy bound is false. One might hope for escapes from one or two of the counterexamples, but it is difficult to see how to escape all of them. The first two are particularly difficult, as it would be hard to accept a universe without either gravitational collapse or inflation. The fact that the strong cosmological entropy bound prohibits either, in principle, means that it must be in conflict with the basic principles of general relativity.

If the strong entropy bound is false, then so must be at least one of the assumptions that went into its derivation. This is why we have been careful to summarize the logic at each step. The list of assumptions that went into it are:

- The first and (generalized) second law of thermodynamics
- Classical and semiclassical black hole thermodynamics.
- The hoop theorem.
- The boundary condition area assumption, eq. (13).
- The strong entropy assumption.

Of these, the boundary condition area assumption is a technical assumption, that helps make the argument cleanly, but if we had to drop it we could still construct the counterexamples. They would just take place within a box, rather than in a cosmological spacetime and so they would then contradict the strong form of the Bekenstein bound. But they would bite no less in that context.

There is a great deal of evidence for classical and semiclassical black hole thermodynamics and we have no independent evidence that the basic principles of thermodynamics are not to be trusted in this regime. The hoop theorem is also well understood and established. The only assumption on this list without independent support is the strong entropy assumption. It must then be wrong.

In fact, as we have emphasized, none of the arguments in the subject provide any independent support for the strong entropy assumption. There is then no argument for the validity of anything stronger than the weak Bekenstein bound and weak cosmological entropy bound.

6 The null entropy bound

We now turn to a different kind of entropy bound bound, proposed by Bousso[8], following a suggestion of Fischler and Susskind[6]. They proposed that the bound restricts the information, or number of degrees of freedom on null, rather than spacelike surfaces bounding the screens.

This principle may be stated as follows.

- **Null entropy bound (Bousso)**. We fix a spacetime manifold $(\mathcal{M}, g)$ on which a quantum field theory has been defined. A *screen* $\mathcal{S}$ will be an oriented two dimensional spacelike surface, possibly open. We then consider one of the four congruences of null geodesics which leave the screen orthogonally, either to the future or the past, and to the left or right of the screen. These may be labeled $L^{\pm}_{l,r}$. We call each a *light surface* associated to $\mathcal{S}$.

- Each of the four light surfaces $L^{\pm}_{l,r}$ may contain a subsurface, $\mathcal{L}$ which satisfies the following condition: The expansion of null rays θ (in the direction going away from $\mathcal{S}$) is non-positive at each point of $\mathcal{L}$. If the boundary of $\mathcal{L}$ contains $\mathcal{S}$ then we call $\mathcal{L}$ a *light sheet* of $\mathcal{S}$. The boundary of $\mathcal{L}$ will generally contain, besides $\mathcal{S}$, a set on which the condition $\theta \leq 0$ fails to hold, either because of the existence of crossing points or caustics, or because the lightsheet intersects a singularity of the spacetime.

- Now, let $\tilde{s}^a$ be the entropy current density of matter, so that

$$S[\mathcal{L}] = \int_{\mathcal{L}} d^3x_a \tilde{s}^a \tag{31}$$

  is the entropy crossing the light sheet. Then the null entropy bound is

$$S[\mathcal{L}] \leq \frac{A[\mathcal{S}]}{4G\hbar} \tag{32}$$

Before turning to its implications, we should mention a possible counterexample, which was proposed by Lowe[26], as its refutation shows the subtlety of the null entropy bound. Consider a box containing a Schwarzchild black hole and thermal radiation, which are in equilibrium at a temperature T. If the box is small enough the ensemble including the black hole has positive specific heat, so the equilibrium is stable. Let us then consider a

spherical spacelike two surface, $\mathcal{S}$ which is a slice of the horizon H of the black hole. Lowe suggests that the horizon H^+ to the future of $\mathcal{S}$ should be considered a light sheet of $\mathcal{S}$. But as the geometry is static, H^+ has no boundary besides $\mathcal{S}$, so that $\int_{\mathcal{H}^+} d^3x_a \tilde{s}^a$ will diverge, since $\tilde{s}^a$ is also constant in equilibrium. Thus, (32) is apparently violated.

The problem, as pointed out by Bousso[8, 9] is that in determining the actual light sheets of $\mathcal{S}$ we cannot use the static geometry, as that is just an averaged description of the actual spacetime geometry. As in any case in which the second law is evoked in a statistical system, we must be careful to take the thermal fluctuations around equilibrium into account. They cause small fluctuations in the spacetime geometry, the result of which is that the actual light sheets $\mathcal{L}$ of $\mathcal{S}$ will not coincide with H^+, when the latter is defined in terms of the average, static geometry. Instead, the small fluctuations will cause parts of the real light sheet to deviate either inside or outside of the averaged horizon. Those that fall inside will shortly hit the singularity (or else, if the singularity is avoided by a bounce, cross, causing caustics.) Those that deviate away from the horizon no longer satisfy the condition that $\theta \leq 0$. Thus the real light sheets will have outer boundaries. Bousso then argues in [9] that the bound will be satisfied.

We may note also that the possible counterexample could be avoided if one only required that the light surface satisfy $\theta < 0$, so as to rule out the marginal case $\theta = 0$. Bousso chooses not to do this as that would eliminate the important example of static black hole horizons.

Finally, we may note that a proof of a closely related conjecture has been given in [11].

As a result, it appears, at least as of this writing, that Bousso's null entropy bound agrees with everything we know. It then may be considered to be a useful, and surprising feature of general relativity at the classical and semiclassical level.

7 Could the null entropy bound extend to quantum gravity?

While the preceding is very satisfactory, we must note that the null entropy bound is formulated in terms of the behavior of the entropy of matter, on a fixed spacetime background. This is already interesting, but for the possible application to quantum gravity we should ask more. This is because general relativity is a dynamical theory, with its own degrees of freedom. We

would then like to know if there is an extension of the bound, even at the classical level, which applies not just to a single spacetime, but to a family of spacetimes which differ by the amplitudes of gravitational waves which may be present in the region containing a screen or a light sheet. If this were the case then a corresponding result would be more likely to hold in quantum gravity, in which there is no fixed spacetime.

Another reason to demand this is that supersymmetry, which seems to be required for the perturbative consistency of quantum gravity, tells us that the distinction between the matter and gravitational degrees of freedom is gauge dependent, and hence not physically meaningful. Furthermore, in perturbative string theory, which seems necessary for perturbative quantum gravity, both the matter and gravitational degrees of freedom arise from excitations of more fundamental degrees of freedom. A bound which requires a strict separation of matter and gravitational degrees of freedom cannot then be formulated in a manner consistent with local supersymmetry and is hence unlikely to extend to supergravity or string theory.

We then investigate, in this section the question of whether there might hold an extension of the null entropy bound that would hold in either of the following cases: i) as a statement on the phase space of general relativity, which allows the fluctuations in the gravitational degrees of freedom to be turned on, ii) in a quantum theory of gravity or iii) in a locally supersymmetric theory. While we do not decide the question, we find that there are two worrying issues, which we now describe.

First problem: Including the gravitational degree of freedom

When the gravitational degrees of freedom are turned on we face a paradox because the light sheets, $\mathcal{L}$ on which the entropy is measured, depend for their definition on the actual values of the gravitational degrees of freedom. This dependence is not weak or gradual, as the positions of the singularities of σ, and hence the location of the boundaries that define $\mathcal{L}$ depend nonlinearly on the values of the gravitational degrees of freedom. Thus, even in classical general relativity, it is difficult to know what would be meant by the null entropy bound once the gravitational degrees of freedom are turned on.

The point may be put the following way. Consider a fixed spacetime $(\mathcal{M}, g_{ab})$, which has a Cauchy surface, Σ, which has embedded in it a screen $\mathcal{S}$ which has a future light sheet $\mathcal{L}$. $\mathcal{L}$ is then to the future of Σ. Now, consider a one parameter family of metrics g^s_{ab}, such that $g^0_{ab} = g_{ab}$ which,

for $s \neq 0$ differ from g_{ab} in a region $\mathcal{F}$ which is to the causal future of a region $\mathcal{R}$ of Σ, not containing $\mathcal{S}$. For each s one can identify the light surface formed by the future null congruence $L(s)$ from $\mathcal{S}$ such that $L(0)$ contains $\mathcal{L}$. In fact, for each s there will be a light sheet $\mathcal{L}(s) \subset L(s)$. Using this one can identify for each s a region $U(s) = L(s) \cap \mathcal{F}$. Let us pick $\mathcal{R}$ such that at $s = 0$, $U(0) \subset \mathcal{L}$.

Now, the notion of a light sheet would be preserved under variations in the gravitational degrees of freedom were it the case that for all s, and all such one parameter families, $U(s) \subset \mathcal{L}(s)$. However it is easy to see that this is not the case[57]. The reason is that one can always find one parameter families g^s_{ab}, specified by initial data in $\mathcal{R}$ such that, for a finite s, all of $U(s)$ will not be in the future lightsheet $L(s)$. The reason is that the gravitational radiation will induce caustics to form in $U(s)$ causing θ to be positive on some part of $U(s)$.

This means that there is no definition of a light sheet which is independent of the initial data in a region $\mathcal{R}$ of a Cauchy surface containing a screen $\mathcal{S}$, even if $\mathcal{R}$ does not contain $\mathcal{S}$. The null entropy bound may hold for each light surface $\mathcal{L}(s)$ in each spacetime g^s_{ab} but there is no extension of the result which holds on the space of solutions or initial data of a spacetime, even when the degrees of freedom are restricted to vary in regions that do not include the screen.

What this means is that one cannot extend Bousso's bound to include the gravitational degrees of freedom in any way which involves defining the entropy of the gravitational degrees of freedom in terms of statistical ensemble of states or histories.

But if this is the case then it is hard to see how there could be an extension of the null entropy bound either to quantum gravity, in which case there is no fixed classical spacetime, or in supergravity, in which there cannot be an invariant distinction between gravitational and matter degrees of freedom.

Second problem: measurability

Another aspect of the problem just discussed is that once the gravitational degrees of freedom are turned on, either classically or quantum mechanically, whether a particular null surface L is a lightsheet of some screen or not depends on the values of the degrees of freedom. This means that there are three choices: I) find a formulation of a cosmological entropy bound that does not require the identification of a lightsheet on which $\theta \leq 0$, II) try to

formulate the condition as an operator equation or III) try to formulate it in terms of expectation values.

The first possibility leads us to the weak entropy bounds, in which the lightsheet plays no role. In case II one must find a set of commuting operators in quantum gravity which are sufficient to define the notion of a light sheet and apply it to their eigenvalues. We must then ask whether the uncertainty principles which arise from the commutation relations of a quantum theory of gravity allow the simultaneous measurement of quantities that must be known to apply the bound. To investigate this we shall assume the standard equal time commutation relations[27, 28, 29]

$$[A_a^i(x,t), \tilde{E}_j^b(y,t)] = \delta^3(x,y)\delta_a^b\delta_j^i \tag{33}$$

where A_a^i is the self-dual connection and $\tilde{E}_j^b$ is the dual of the pull back of the self-dual two form, Σ in any spacelike surface (and all other commutators vanish). We may note that these hold in a large class of theories, including all the extended supergravities, as they arise from the generic form[28]

$$I = \int_{\mathcal{M}} \Sigma^{AB} \wedge \dot{A}_{AB} + \dots \tag{34}$$

It is difficult to imagine that these do not hold in the effective field theory which is the low energy limit of string theory or whatever the true quantum theory of gravity is.

It is not hard to show that,

- If $\theta^{\pm}(s)$ is the expansion of the future and past going null geodesics normal to a two surface $\mathcal{S}$ at a point $s \in \mathcal{S}$ and $A[\mathcal{S}]$ is the area of $\mathcal{S}$ then[30]
$$[\theta^{\pm}(s), A[\mathcal{S}]] = 0 \tag{35}$$

- Let (s,u) be coordinates on a null surfaces, L, generated by the null geodesics leaving $\mathcal{S}$ orthogonally, where s labels a congruence of null geodesics and u is an affine parameter along each geodesic such that $\mathcal{S}$ is defined by $u=0$. If $\theta^{\pm}(s,u)$ is the expansion of the congruence at (s,u) then, for $u \neq 0$ we have,
$$[\theta^{\pm}(s,u), A[\mathcal{S}]] \neq 0 \tag{36}$$
$$[\theta^{\pm}(s,u), \theta^{\pm}(s)] \neq 0 \tag{37}$$

Thus, there does exist a basis which comes from simultaneously diagonalizing the expansion of the congruence of null rays at a screen, and its area. But in such a basis the operators $\theta(s, u)$ off the screen are not diagonal. Thus, we cannot identify the outer boundary of the lightsheet in any basis in which we can identify a screen and measure its area. This suggests that there cannot exist an operator form of the null entropy bound.

The remaining possibility is to formulate the condition for a screen in terms of expectation values. Presumably this should be possible in the semiclassical limit, otherwise the null entropy bound could not be true in that limit. I am not aware of any proposal to implement it beyond the limit. One way to see why this is unlikely to work is to discuss how it would have to work in a path integral formulation of the theory.

Let us first note that to be relevant to the null entropy bound, a histories formulation will have to be formulated in terms of causal histories, as there are no analogues of light sheets in Euclidean metrics. Fortunately, there are now non-trivial proposals[31, 32, 33, 34, 35, 36, 37] and results[38, 39] concerning formulations of quantum gravity in terms of causal,lorentzian path integrals. In such a causal histories formulation, each history in the sum over histories comes with its own causal structure. The problems we are discussing can then be stated as follows: it may be possible to pick out consistently a set of histories in which the area of a preferred family of surfaces and the expansions of null geodesics at those surfaces are given and fixed. But in these histories the properties of the light surfaces generated by following null geodesics from the screens cannot be controlled, and will fluctuate as the sum over histories is taken. Since caustics and other singularities are generic for such surfaces, the observer at the screen will be unable to control or measure any variables at the screen to prevent the formation of singularities in the light surfaces of the histories.

Another way to say this is that the degrees of freedom of the light surface include components of the metric and connection on the light surface itself. Singularities and caustics will form for generic values of these parameters, but where they form varies as the degrees of freedom fluctuate. One cannot then consistently count the number of degrees of freedom on the non-singular part of the light surface, this involves a logical contradiction since the presence and location of the singularities itself depends on the values of the fluctuating degrees of freedom.

Before closing this discussion, we note that our argument does not exclude one radical possibility, which is that there is an entropy associated with single classical configurations of the gravitational field. This has been

suggested by Penrose[40], and there are some old arguments for it, which rely on the impossibility of either building a container to constrain gravitational radiation, inducing a gravitational ultraviolet catastrophe or measuring the pure state of gravitational radiation[41]. In the present context this would suggests that $\int_{\mathcal{L}} \sigma^{ab}\sigma_{ab}$, where σ is the shear of the null congruence, might be taken as a measure of the gravitational entropy on a lightsheet $\mathcal{L}$ in a single classical spacetime.

This is an intriguing possibility, which deserves investigation. It does not affect the following considerations, as the difficulties we find with a null form of the holographic principle would not be lessened.

Part II
HOLOGRAPHIC PRINCIPLES

A holographic principle is meant to be a formulation of the dynamics of a quantum theory in terms of its screen, or boundary hilbert spaces. A holographic principle requires some form of an entropy bound, but it requires also that the dynamics of the theory can be formulated entirely in terms of the degrees of freedom measurable on the screen.

There are different kinds of holographic principles, corresponding to the different possible kinds of entropy bounds. We consider in turn, strong, null and weak forms of the holographic principle.

8 The strong holographic principle

The classic formulation of the strong holographic principle is meant to apply to the case we discussed in section 2. We have a quantum or classical spacetime, $\mathcal{M}$, with a boundary $\partial\mathcal{M} = R \times \mathcal{S}$, where R corresponds to the time coordinate. One then postulates boundary and bulk algebras of observables, $\mathcal{A}_S$ and $\mathcal{A}_{bulk}$ and $\mathcal{H}_S$ and $\mathcal{H}_{bulk}$ as in section 2. In addition one specifies on each hilbert space a hermitian hamiltonian $h_{\mathcal{S}}$ and h_{bulk}. In a gravitational theory this may require the specification of gauge conditions on the boundary, in this case there is a family of bulk and boundary hamiltonians which depend on the gauge conditions. The principle is then formulated as follows

- **Strong holographic principle** There is an isomorphism

$$\mathcal{I} : \mathcal{H}_{bulk} \leftrightarrow \mathcal{H}_S \tag{38}$$

such that $\mathcal{I} \circ h_{bulk} = h_{\mathcal{S}}$.

Can this principle be satisfied? There are two cases: gravitational theories and non-gravitational theories.

Failure of the strong holographic principle in gravitational theories

By a gravitational theory we mean here theories in which the gravitational degrees of freedom are allowed to fluctuate, so that any principle must hold for all the possible initial data that the theory allows. This will be specified classically or semiclassically by initial data on Σ or quantum mechanically by the specification of a state in $\mathcal{H}_{bulk}$. We do *not* mean quantum field theory on a particular fixed spacetime metric g_{ab} on $\mathcal{M}$.

There is some evidence that the strong holographic principle may hold in a quantum theory of gravity. One piece of evidence comes from canonical quantum general relativity, with a non-zero cosmological constant, and certain boundary conditions, called the Chern-Simons boundary conditions[13, 14, 15]. In this case the boundary Hilbert space is found to be of the form

$$\mathcal{H}_{\mathcal{S}} = \sum_a \mathcal{H}^a_{\mathcal{S}} \tag{39}$$

where a is an eigenvalue of the area operator, which is known from [23, 24] to have a discrete spectrum. Each of the eigenspaces $\mathcal{H}_{\mathcal{S}}$ is a space of $SU_q(2)$ intertwiners on a punctured S^2 where the labels on the punctures, which are taken from the representations of $SU_q(2)$ are related to the area[13, 14]. The level k is related to the cosmological constant by[13], $k = 6\pi/G^2\Lambda$. The dimension of the space of intertwiners does satisfy eq. (6), with a renormalization of Newton's constant defined by $G_{ren} = cG_{bare}$, with $c = \sqrt{3}/\ln(2)$. Thus, it is clear that the weak Bekenstein bound is satisfied.

There are also some results concerning the bulk Hilbert space. Before the hamiltonian constraint is imposed, an infinite dimensional space of bulk states can be identified for each a, which has an orthonormal basis given by the distinct embeddings (up to diffeomorphisms) of quantum spin networks in the bulk, whose edges meet the boundary at the punctures. One can then show that there is, for each set of punctures, a finite dimensional space of solutions to the Hamiltonian constraint which is isomorphic to the corresponding boundary Hilbert space[13]. These are constructing by moding out the infinite dimensional kinematical Hilbert spaces by a set of equivalence

relations which generate the recoupling identities of quantum spin networks. It is known that for a certain class of states these recoupling identities realize the action of the Hamiltonian constraint[13].

In this case the strong entropy assumption then comes down to the conjecture that these provide a complete set of solutions to the Hamiltonian constraint in the bulk. There is presently no evidence either way on the correctness of this conjecture. It is attractive to argue that if it is true we have quantum general relativity in this particular case expressed in closed form as a theory which satisfies the strong holographic principle. We may also note that these results all hold for both the euclidean[13] and lorentzian[14] cases as well as for supergravity[15].

However, if we accept the conclusion of the previous arguments, then this conjecture must in fact be false. We have found instead that for the case of a gravitational theory the strong holographic principle cannot hold unless the boundary of the spacetime has either infinite or indeterminate area. The reason is that, as we have shown, the strong entropy assumption, and the strong forms of the Bekenstein bound and the cosmological entropy bounds all fail. As a result we cannot assume that the bulk and boundary Hilbert spaces have the same dimension. However, we have also shown that the weak Bekenstein and weak cosmological entropy bounds hold, which means that $\mathcal{H}_{\mathcal{S}}$ is finite dimensional or, generically, is composed of finite dimensional subspaces, which are the diagonal sectors of the area $A[\mathcal{S}]$.

Since no bound has been found to hold which restricts the dimension of $\mathcal{H}_{bulk}$ there are two possibilities, either it is infinite dimensional, or it does not exist. In the latter case there is nothing for an isomorphism to map the boundary state space to. If it is infinite dimensional it could only be mapped to the boundary which has either indeterminate or infinite area. Thus we conclude that a form of the strong holographic principle could only hold in those cases.

Let us now consider the case of indeterminate area more closely. Let us consider the Schroedinger picture operators, defined in terms of the time at the boundary. It is clear that by causality $\hat{A}[\mathcal{S}]$, the operator that measures the area of the boundary must commute with h_{bulk}, as the latter is a function only of degrees of freedom in the bulk of Σ which are causally unrelated to degrees on $\mathcal{S}$. Another way to say this is that where we must be able to move the boundary locally, thus changing its area, without affecting the physics in regions of the bulk causally disconnected from the events of moving the boundary. Since $[\hat{A}[\mathcal{S}], h_{bulk}] = 0$, we must be able to construct projection operators in the bulk, $\hat{P}_a$ corresponding to every eigenvalue, a in the spec-

trum of $\hat{A}[\mathcal{S}]$, and define the restricted Hamiltonian $h^a_{bulk} = \hat{P}_a h_{bulk} \hat{P}_a$. By causality it must then be the case that the strong holographic principle work between the corresponding subspaces $\mathcal{H}^a_{\mathcal{S}}$ and $\mathcal{H}^a_{bulk}$ for *each* value of a.

But now we can apply the argument for the case of a finite area boundary. Since there is no bound on the dimension of $\mathcal{H}^a_{bulk}$, it must have infinite dimension, thus it cannot be isomorphic to $\mathcal{H}^a_{\mathcal{S}}$.

This leaves only the case that the boundary has infinite area. But in this case there cannot be a cosmological version of the principle, as generic spatial regions in generic cosmological solutions have finite area boundaries. Thus, at best, the strong holographic principle could only apply to the case of non-compact spacetimes with boundary.

This may be satisfactory, but it comes with a price, which is that we will not be able to apply the principle to any case in which the boundary is moved inside the non-compact spacetime to coincide with a finite area surface. This means that for any such surface, labeled again by its area, a the holographic correspondence (38) will map all but a finite dimensional subspace of $\mathcal{H}^a_{bulk}$ to degrees of freedom that are contained within $\mathcal{H}^\infty_{\mathcal{S}}$, but are not representable within $\mathcal{H}^a_{\mathcal{S}}$. This must hold for any finite a. It means that for no finite a can there be any correspondence between $\mathcal{H}^a_{bulk}$ and $\mathcal{H}^a_{\mathcal{S}}$, as almost all of the information in the former is not representable in the latter. This is counterintuitive, it means no matter how far we move the boundary out, the representation space of the boundary observables do not capture most of the information about the bulk observables, so long as the area of the boundary is finite.

This would be very disappointing, what it really means is that there is no way going to an infinite boundary can save the situation, once it is realized that for any finite area boundary there can be no holographic isomorphism (38). Thus, we conclude that there cannot be an implementation of the strong holographic principle in a gravitational theory.

Realization of the strong holographic principle in non-gravitational theories

It is surprising, and striking, that in spite of its failure for gravitational theories, there are realizations of the strong holographic principle for non-gravitational theories. These occur in a special case which is Anti-DeSitter backgrounds in $D+1$ dimensions[16, 17, 18]. In these spacetimes the asymptotic boundary is timelike and is in fact conformally compactified Minkowski spacetime (CM) in D dimensions. The existence of such a correspondence

was conjectured first by Maldacena in a string related argument[16, 17], but has since been shown to hold quite generally for non-gravitational theories on AdS backgrounds[18]. There is in fact a rigorous theorem in axiomatic quantum field theory that shows that gives such an isomorphism for generic field theories on AdS spacetimes[18].

The reason behind this correspondence is clearly that $SO(D, 2)$ acts as the symmetry group on AdS_{D+1} and as the conformal symmetry group of CM_D. As a result one can establish an isomorphism (38) for general quantum field theories on AdS backgrounds.

We can also see why the argument given just above is superseded in the AdS case. A key fact is that AdS spacetime has no Cauchy surface. The reason is that the evolution in the bulk requires the specification of data on the timelike asymptotic boundary of the spacetime. If the boundary fields are not specified there is no deterministic evolution for the bulk degrees of freedom. As a result the boundary degrees of freedom are part of a complete specification of the dynamics of the bulk theory. This makes it less surprising that the dynamics can be reduced to a description of boundary degrees of freedom, in this case the bulk to boundary map is plausibly a reduction to the data necessary to determine a solution, and may play a role similar to that of the map that relates a solution to boundary data in spacetimes with Cauchy surfaces. This is very different from what happens in asymptotically flat spacetimes in which only the only quantities measurable at spatial infinity are a finite set of conserved quantities.

The key question is then whether there are conformal quantum field theories for general D on CM_D. There are certainly free field theories, for which the correspondence holds[18]. It may then be expected to hold also for interacting theories in those special cases in which there is a conformal quantum field theory on CM_D. There is evidence that one such case is $N = 4$ supersymmetric Yang-Mills theory for $D = 4$[16, 17]. By the general arguments of [18] one would expect there to exist on AdS a supersymmetric theory, whose spectrum transformed under a supersymmetric extension of $SO(4, 2)$, with 16 supercharges. There is a great deal of evidence that this is the case, at least in the limit $N \to \infty$. In this case the theory appears to be the weak coupling limit of supergravity compactified on an $AdS_5 \times S^5$ background.

The big question

We then have the following question. We have argued that the strong holographic principle cannot hold in a gravitational theory. It can hold in a quantum field theory on a fixed background, and indeed in the particular case of AdS spacetimes it seems to be a generic feature. But it should be expected to break down as soon as the gravitational degrees of freedom are turned on[9]. More precisely, we expect our first two counterexamples to arise as soon as either gravitational collapse or inflation could occur in regions of the bulk.

At the same time, in the particular case of $AdS_5 \times S^5$ the isomorphism seems to exist and the bulk theory is then the weak field limit of a gravitational theory. What then happens when the gravitational constant is turned up, so that the gravitational degrees of freedom are excited? This is the big question. There seem to be three possibilities:

- **1** Something is wrong with the above reasoning, at least in the case of supersymmetric theories.
- **2** In supergravity or string theory on spacetimes which are asymptotically $AdS_5 \times S^5$ the counterexamples cannot arise. This means that there cannot be small black holes that form from gravitational collapse and there cannot be any possibility of choosing the initial conditions in the interior so as to drive the theory into an inflating phase.
- **3** The correspondence holds at the level of the background dependent quantum field theory defined on $AdS_5 \times S^5$ by the weak coupling limit of supergravity or string theory, but breaks down as soon as the gravitational constant or the initial data is large enough that strong gravitational fields can arise.

The first possibility is of course always there, this is why we have been very careful to keep track of the logic leading to the conclusion that the strong entropy assumption, strong Bekenstein bound and strong cosmological entropy bounds must all be false in gravitational theories. If there is an

[9] The same questions can also be asked in the $2+1$ dimensional case, where the AdS/CFT correspondence has also been worked out[17]. However, given that quantum gravity and supergravity in $2+1$ dimensions are topological quantum field theories, there may be little to learn from this case that is generally useful. TQFT's are by definition theories whose observables and states are defined on boundaries of the spacetime.

error it must be either in the reasoning or in the unexpected failure of one of the other assumptions listed in section 5.

The second possibility seems unlikely, and in any case were it true it would mean that this particular case is non-generic in ways that suggest it is not a very good example of a quantum theory of gravity.

We must then ask if there are any results that contradict the third possibility. At of this writing all of the results found which support the conjecture in the $AdS_5 \times S^5$ case relate boundary observables of supergravity on that background to expectation values of the supersymmetric Yang-Mills theory. While the construction of representatives in the Yang-Mills theory of bulk observables in the bulk theory have been discussed, there is so far no calculation which gives a non-trivial test of these correspondences. It is also the case that most, if not all, of the calculations of N-point functions which support the conjecture are in any case forced by the action of the supersymmetry group. It then seems to be the case that even if it disagrees with some interpretation of the conjectured correspondence, there are no actual results which so far contradict possibility **3.**

There is a final remark which is consistent with this third possibility which is the following. Gravitationally bound systems including black holes have generically negative specific heat. However, the positivity of the specific heat for an equilibrium ensemble is guaranteed for any system defined by a partition function. In particular, the thermal quantum field theory gotten by raising the temperature in the $N = 4$ supersymmetric Yang-Mills theory is defined by a partition function. Therefor all equilibrium configurations will have positive specific heat.

We can then ask how configurations such as a system of planets or small black holes in the bulk of the AdS spacetime are to be represented in terms of states of the $N = 4$ supersymmetric Yang-Mills theory. There are two possibilities: this can be done, but involves configurations that are sufficiently far from equilibrium in the Yang-Mills theory that they cannot be described by a partition function. Or, the correspondence breaks down as soon as gravitationally bound states of the bulk theory arise whose statistical ensembles have negative specific heat.

As a final remark, we note that S duality is still a conjecture, outside of the BPS sector of either $N = 4$ superYang-mills theory or string theory. Thus, if the isomorphism (38) fails beyond the BPS sector there is nothing that constrains S-duality to hold on both sides of it.

What about the black hole information paradox?

One reason that the strong holographic principle has been advocated by some people is that it guarantees a solution to the black hole information paradox. Thus, one can wonder if there is an independent argument for the strong holographic principle which follows from the possibility that it is necessary to give a consistent resolution of the black hole information paradox.

The answer is negative, because a large part of the black hole information paradox depends on the strong entropy assumption, which we have found is false. Once it is realized that the strong entropy assumption is false, there is no reason to presume that the amount of information measurable by observers in the interior of the black hole horizon is constrained by the black holes's horizon area. One can then imagine that an arbitrarily large amount of information may be stored in the region to the future of the horizon independent of its surface area.

One very plausible scenario, which is supported by several semiclassical calculations, is that there is a bounce as the collapsing star nears what would be the classical singularity, leading to the formation of a new expanding region of spacetime which could contain an arbitrarily large amount of information (measured from the point of view of internal observers.) Given the possibility of making a transition back to an inflationary phase this region could resemble our universe.

What will then happen when the horizon evaporates. In such a case there is no real spacetime singularity, and there is correspondingly no need for an event horizon. This means that attempts to construct a paradox by making small perturbations to the usual black hole global structure, which do not eliminate the singularity, are likely of no relevance to the real physical problem. There will be an apparent horizon and under evaporation it will shrink to a size at which quantum fluctuations of the gravitational field will be significant. At this point one will have a small wormhole, linking our spacetime to the origin of a large inflating region. Most of the information that went into the black hole will be trapped in the new region, but there will be no local violation of any physical principle. This does not mean that there cannot be global unitary evolution in the whole spacetime, but only that not all measurements made in the interior of the bulk can be communicated to null infinity.

Is this kind of scenario plausible? This is one of the key questions as we investigate what form a holographic principle could take in a gravitational

theory, in which only the weak and null cosmological entropy bounds survive.

9 The null holographic principle

If we give up on the possibility of a strong holographic principle that could hold in either a gravitational or cosmological theory, we are forced back to the next strongest possibility, which is to construct a form of the holographic principle which would extend the null form of the cosmological entropy bound proposed by Bousso. Since that bound is only known to hold at the semiclassical level in a fixed cosmological spacetime $(\mathcal{M}, g_{ab})$, let us ask what form such a null holographic principle would have to take in this case.

The problem is clearly to find a collection of light sheets that cover the spacetime so that the evolution of matter fields may be described in terms of them. What is needed can be defined as follows.

- A classical spacetime $(\mathcal{M}, g_{ab})$ has a *single null holographic structure* if there exists a one parameter (continuous or discrete) family of screens $\mathcal{S}(t)$ with a corresponding one parameter family of light sheets $\mathcal{L}(t)$, (each possibly made by joining two lightsheets of $\mathcal{S}(t)$), such that for any two times, s and t, the classical or quantum state of the matter on $\mathcal{L}(s)$ is completely determined by that on $\mathcal{L}(t)$. In the quantum mechanical case, this means there is a one parameter family of Hilbert spaces, $\mathcal{H}(s)$, which satisfies the bounds

$$dim\mathcal{H}(t) \leq e^{A[\mathcal{S}(t)]/4G\hbar} \tag{40}$$

  such that there is for each s and t a unitary operator

$$U(s,t) \circ \mathcal{H}(t) = \mathcal{H}(s) \tag{41}$$

This is the minimal requirement, if there is going to be a representation of the quantum dynamics of matter in the spacetime $(\mathcal{M}, g_{ab})$ that captures the basic principles of ordinary quantum mechanics.

The problem is that such a structure does not exist for generic spacetimes $(\mathcal{M}, g_{ab})$. By (40) and (41) we see that all the screens in the family must have the same area, otherwise their Hilbert spaces cannot be unitarily equivalent. The problem is that in generic spacetimes the lightsheets of any single screen will not cover the complete future or past of any Cauchy surface. The reason

is that the lightsheets are compact, and of limited extent. This is in fact the whole point of Bousso's bound. Consequently, given any two screens $\mathcal{S}(s)$ and $\mathcal{S}(t)$ it will almost never happen that the corresponding light sheets $\mathcal{L}(s)$ and $\mathcal{L}(t)$ form a *complete pair*. By a complete pair[42, 43, 44] is meant a pair of non-timelike surfaces such that $\mathcal{L}(s)$ is within the causal future of $\mathcal{L}(t)$ and is complete in that no event can be added to $\mathcal{L}(s)$ which is also in the causal future of $\mathcal{L}(t)$, which is acausal to $\mathcal{L}(s)$, and the same is true reversing s and t and past and future.

It is only between complete pairs that one can expect to find deterministic evolution in either a classical or quantum theory on a fixed spacetime.

In a few very special cases involving highly symmetric spacetimes, one can find such a single null holographic structure[8, 9]. But these are special cases in which the symmetry allows the lightsheets to be complete futures of Cauchy surfaces. One can say that in highly symmetric spacetimes such as Minkowski or DeSitter spacetime complete lightsheets can exist because by the symmetry there is so little information for an observer in the spacetime to measure. Once any inhomogeneity is turned on we expect that the light surfaces will contract to finite regions and any two will be very unlikely to make a complete pair. But what is required generically is not only that we have a family of light surfaces any two of which make a complete pair. In addition the screens of those light surfaces must all have the same area. There is no reason to believe these conditions can be satisfied in a generic spacetime.

Can we weaken the condition? We can if we give up the idea that there is a one parameter family of light surfaces, each of which has a Hilbert spaces, all of which are unitarily equivalent. This idea conserved the structure of ordinary quantum mechanics in which there is a single Hilbert space on which evolution is unitarily implemented. However it is clearly ruled out.

For a generic spacetime it is clear that the lightsheets of no screen will be complete in the future or past of any Cauchy surface. In this case if we want a description in terms of screens we must allow the possibility that a complete description of the system will require generally more than one screen, representing information available to different local observers in the spacetime. This means that a complete holographic description of a quantum field theory in a cosmological spacetime system will generically involve multiple Hilbert spaces, each of which represents information available to observers at different screens. Time evolution must then be represented in terms of maps between density matrices in these HIlbert spaces. Unitary evolution will only be possible for pairs of such Hilbert spaces that describe

causal domains that form complete pairs.

Is such a multiple Hilbert space description of a quantum theory in a cosmological spacetime possible? In fact exactly such a structure was proposed in [42, 43, 44], under the name of quantum causal histories. It arose from an independent line of thought, coming from attempts to take seriously the limitations on the algebra of observables coming from the causal structure of relativistic cosmological theories. As we showed in [10] this structure does admit a formulation of a weak holographic principle.

10 Is every two surface a screen?

In some approaches to the cosmological holographic principle, screens are two surfaces satisfying special conditions. Such conditions are also used to distinguish which side of a two-surface may be a screen, for example for screens in normal regions in Bousso's approach, only one side of a surface will in general be a screen.

It is then important to ask whether any such conditions may be imposed in the case of quantum cosmology. There seems to be a problem with each of the possible conditions that have been offered at the semiclassical level.

- As there are no asymptotic regions, and no boundaries to a cosmological spacetime, there are no global event horizons. Generic spacetimes do not generically contain any single null holographic structures, which means that the information measurable on any screen cannot be used to completely specify the state of a classical or quantum cosmology, and a holographic principle cannot be formulated in terms of any single one parameter family of screens.

- The operator that measures the convergence of null rays at a surface does commute with the operator that measures its area. This could be used in a quantum theory of gravity to distinguish the two sides of a screen. However, this does not have the same implications in the quantum theory because the local positive energy conditions on the energy momentum tensor do not hold even at the semiclassical level. Because of this null rays may diverge after beginning to converge and trapped surfaces cannot be distinguished by any local conditions. Furthermore, the operator that measures the convergence of a null ray a finite distance from the surface does not commute with its convergence

on the surface. This means that in a quantum theory one cannot apply the tests we use in the classical theory to pick out a lightsheet. Consequently there seems to be no reason in the quantum theory to choose one side of a screen over another.

- No condition can be imposed having to do with the volume of a spacelike region bounding a screen, for the volume of a region is measured by a quantum mechanical operator[24, 25]. Generic states will be superpositions of eigenstates of the volume operator for any region. One can thus not require that the side of a two-surface which encloses the smallest volume is a screen.

- As we see from several of the counterexamples, there is no paradox in considering both sides of a surface to be a screen, so long as one understands the entropy bound weakly, so that it applies only to information gained by making measurements of fields at the surface, which may or may not allow deductions to be made concerning the density matrix or state to the causal past of the surface.

If there is no criteria which can be applied in a quantum theory of cosmology to pick out which surfaces are screens, or to pick one of the two sides of a two-surface to serve as a screen, then we must conclude that every two-surface may be a screen, and the opposite side of any screen may also be a screen. In the quantum theory one may still make observations on a screen, but one will not in general be allowed to deduce anything about the extent to which those observations allow a complete description of the physics on a finite lightsheet. Since that was the reason to prefer one screen over another, the conclusion is that in the quantum theory if a screen is a useful concept, then all two surfaces may be screens.

This conclusion will play an important role in the weak holographic principle because it means that in a quantum theory we may use the properties of a screen as a place where measurements may be made to constrain, or even define,its geometrical properties, rather than the reverse, which is what we do in the semiclassical theory.

11 Conclusions reached so far

To motivate the weak form of the holographic principle we summarize the results of the argument so far.

- The strong entropy conjecture is apparently false, which means that the weak, rather than the strong version of the Bekenstein bound is true.

- The strong cosmological entropy bound is false.

- The null cosmological entropy bound cannot be formulated in a quantum theory of gravity once the gravitational degrees of freedom are turned on, at least in the conventional terms in which entropy is related to the lack of purity of density matrices.

- The weak cosmological entropy bound may be satisfied in a quantum theory of gravity. This is formulated as a relationship between the information capacity of a screen $\mathcal{S}$, as measured by the dimension of the Hilbert space $\mathcal{H}_{\mathcal{S}}$ which provides the smallest faithful representation of the algebra of observables $\mathcal{A}_{\mathcal{S}}$ on the screen, and its area $A[\mathcal{S}]$.

- From the wiggly surface problem we learn that the appropriate measure of the area of a screen is not the area of $\mathcal{S}$. Instead, the amount of information that can be stored on any screen, $\mathcal{S}$ is bounded by the minimal area of the cross sections of congruences of light rays that intersect $\mathcal{S}$. This means that a causal structure is required in order to make sense of a holographic bound in a quantum cosmological theory.

- The information coded on a screen $\mathcal{S}$ then concerns its causal past. But it then follows that in most histories there will be no single screen on which a complete description of the universe may be coded, for there will, in the classical limit, be generally no spacelike two-surface such that the past of its lightsheets contains a Cauchy surface. (We see this also from the throat and inflationary examples.) From this it follows that a holographic description in a quantum cosmology must involve many screens $\mathcal{S}_i$, and that the information available at any one screen will almost always be incomplete.

- One implication of this is that the most complete description of the quantum state available on any single $\mathcal{S}_i$ must be a density matrix ρ_i on $\mathcal{H}_i$. This is because there will in general be quantum correlations that connect measurements made on $\mathcal{S}_i$ with degrees of freedom that are recorded on other surfaces $\mathcal{S}_j$.

12 The weak holographic principle

A weak form of the holographic principle must be consistent with these conclusions. One possible form, which is, is that given in [10]. In somewhat less technical language than that give there, the principle holds that

1. A holographic cosmological theory must based on a causal history, that is, the events in the quantum spacetime form a partially ordered set under their causal relations.

2. Among the elements of the quantum spacetime, a set of screens can be identified. A screen $\mathcal{S}$, is a 2-sided object, which means that it consists of a left and right side, each of which has a distinct past and future, but such that the past right side is to the immediate past of the future left side, and visa versa.

3. Associated to each side of the screen, labeled L, and R are an algebra of observables, $\mathcal{A}_{\mathcal{S}}^{L.R}$ each of which is represented on a finite dimensional Hilbert space $\mathcal{H}_{\mathcal{S}}^{L,R}$. The observables in $\mathcal{A}_{L.R}$ describe information that an observer at the screen may acquire about the causal past of one side of the screen, by measurements of fields on that side of the immediate past of the left or right side of the screen.

4. $\mathcal{H}_{\mathcal{S}}^{L} = \mathcal{H}_{\mathcal{S}}^{R\,\dagger}$, which means they have the same dimension.

5. All observables in the theory are operators in the algebra of observables $\mathcal{A}(\mathcal{S})$ for some screen $\mathcal{S}$.

6. The area of a screen $\mathcal{S}$ is *defined* to be

$$A[\mathcal{S}] \equiv 4G\hbar \ln Dim\,(\mathcal{H}_{\mathcal{S}}) \qquad (42)$$

More discussion of this principle may be found in [10]. Its message is that *all* observables in a quantum theory of cosmology are associated with two-surfaces, and represent information reaching a surface from its causal past. Besides the logic we have followed here, there are two sets of arguments that might be used to support this hypothesis.

Quasi-local quantities in classical general relativity

Even in classical general relativity, it is well understood that diffeomorphism invariance and the equivalence principle forbid the possibility of local definitions of the basic dynamical quantities such as energy, momentum and

angular momentum. These kinds of quantities can only be defined in terms of integrals over two dimensional surfaces in the spacetime. When those surfaces are taken to the boundary, in non-cosmological spacetimes, these become the well known asymptotic definitions of energy, momentum and angular momentum. However, even in cosmological spacetimes where there are no boundaries one may define what are called *quasi-local* observables[45, 46], in which the energy, momentum and angular momentum of an arbitrary region are defined in terms of certain integrals over its boundary. Since Penrose's original suggestion[45] many different proposals have been made for such quasi-local observables[46].

If there are to be non-trivial notions of energy, momentum and angular momentum in a quantum theory of cosmology then, these must be defined so that their classical limits are these quasi-local quantities. The simplest possibility is that the hamiltonian in quantum gravity should itself be quasi-local, that is defined on two dimensional surfaces, which in the classical limit will become spacelike surface embedded in spacetime. This implies some form of the holographic principle, for if the Hamiltonian is associated with surfaces there must be many hamiltonians, each associated with a different choice of surfaces, and the same must be true of the algebra of observables and the hilbert spaces on which they are represented.

Relational approaches to quantum cosmology

Another kind of argument for the importance of surface observables in a quantum theory of cosmology was given by Crane[3], even before the holographic hypothesis of 't Hooft and Susskind was proposed. Crane noted the difficulties of defining a coherent measurement theory for a quantum state "of the whole universe" and proposed instead that the division of the universe into two parts-system and observer-that is basic to Bohr and Heisenberg's measurement theory might be relativised, so that there would be not one quantum state of the universe, but a system of observable algebras and hilbert spaces, one associated with every possible splitting of the universe into two parts[3].

To realize this idea, Crane proposed a categorical framework to describe the association of Hilbert spaces with boundaries. This was based on positing functorial relationships between the category of cobordisms of manifolds and the category of Hilbert spaces[3]. These structures are closely related to topological quantum field theory, as those theories can be formulated in such categorical terms. As topological quantum field theories are the only

class of field theories that naturally yield finite dimensional Hilbert spaces, one may try to use them to construct examples of holographic theories[13]. Furthermore, as Crane pointed out, it may be possible to extend these structures to quantum theories of gravity because it is a fact that at both the classical and quantum mechanical level, and for any dimension[47], general relativity and supergravity can be understood as deformed or constrained topological quantum field theories[49, 23, 13, 19, 51, 50, 14, 15].

Crane's proposal has been an inspiration for the development of what have been called relational[48, 42, 43, 44] or pluralistic [19, 5] approaches to quantum cosmology. Using the fact that general relativity and supergravity are constrained topological field theories, it has been possible to realize this idea in the context of full formulations of quantum gravity and $\mathcal{M}$ theory [33, 34, 54].

An even stronger version of Crane's argument was proposed recently by Markopoulou[42, 43, 44], who noted that *even in classical general relativity* the logic of propositions which can be given truth values by observers in a closed universe is non-boolean, because each observer can only assert the truth of falsity of propositions about their past. Rather than being a boolean algebra, the algebra of propositions relevant for a classical cosmological theory is a multivalued Heyting algebra[42]. When quantized, the resulting algebra of projection-like operators cannot be represented on a single Hilbert space, instead, it requires a collection of Hilbert spaces, one for every possible event at which observations are made[43]. As each observer receives information from a distinct past, the algebra of observables they can measure, and hence the Hilbert spaces on which they represent what they observe, must vary[10]. Given the conclusions reached in the preceding sections of this paper, this is framework is then appropriate for a formulation of the weak holographic principle[10].

13 Conclusions

The conclusion of the arguments we have given here is that the holographic bound and holographic principle can only survive in a quantum theory of cosmology in their weak forms, proposed in [10]. While logically weaker, this form is more radical than the strong forms, in its implications for how

[10] Related structures have been studied also by Isham and collaborators [55], who note that structures built of many Hilbert spaces can be used to formulate the consistent histories proposal[56] precisely.

a measurement theory of quantum cosmology must be constructed. First, the weak forms require that causal structure exist even at the Planck scale. This most likely cannot be realized in a conventional formulation of quantum cosmology in which the observables of the theory act on a single Hilbert space containing the physically allowed "wavefunctions of the universe." Instead, such a description may have to be formulated along the lines proposed in [42, 43, 44] in which there is a network of Hilbert spaces, each providing a representation for an algebra of observables accessible to a single local observer at an event or a local region of a spacetime history. These will be related to each other by maps which reflect the quantum causal structure.

In such a spacetime, evolution becomes closely intertwined with the flow of quantum information which also defines the causal structure at the Planck scale. Interactions have to do with the processing of the information at events; as noted in [43, 44] a quantum spacetime then becomes very like a quantum computer that can dynamically evolve its circuitry.

It is then difficult to escape the conclusion that the holographic principle, in its weak form, is telling us that nature is fundamentally discrete. The finiteness of the information available per unit area of a surface is to be taken simply as an indication that fundamentally, geometry must turn out to reduce to counting. Of course this conclusion has been reached independently through other arguments coming from quantum gravity[1, 23, 24, 25, 5] and string theory[52, 2]. But, as can be seen most clearly from the argument of Jacobson[53], the entropy bounds and holographic principle tell us that the description of nature in terms of classical spacetime geometry is not only analogous to the laws of thermodynamics, it must be exactly the thermodynamics of the fundamental discrete theory of spacetime.

What we learn from the analysis of this paper is that in such a theory there is no room for the notion of a bulk theory, and hence no fundamental role for a bulk-boundary correspondence. There is instead a network of screen histories, which describe what possible observers might be able to observer from particular events in their spacetime. By averaging over histories a bulk description may emerge at the semiclassical level, but only as an approximation in which the past of a particular observer can be described to first order in a perturbation expansion in terms of a particular fixed classical history. Thus the proper role of a bulk-to-boundary map may be to serve as a correspondence principle to constrain the classical limit of a background independent quantum theory of gravity.

To put it most simply: the holographic principle is not about a relationship between two sets of concepts, bulk and screen and geometry and

information flow. It is the statement that the former reduce entirely to the latter in exactly the same sense that thermodynamic quantities reduce to atomic physics. The familiar picture of bulk spacetimes with fields and geometry must emerge in the semiclassical limit, but these concepts can play no role in the fundamental theory.

Can this picture be used to construct a realistic quantum theory of gravity which addresses also the other problems in the subject? As mentioned in [10] an example of such a theory is provided by a class of background independent membrane theories proposed in [33]. These extend the formalism of loop quantum gravity in a way as to provide a possible background independent form of string theory[34, 54]. So the answer is a very provisional, yes. Much work remains to be done, but the moral is that the holographic principle, in at least its weak form, is likely to feature significantly in both the mathematical language and the measurement theory of the future background independent quantum theory of gravity.

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